NPS-57HW73011A

NAVAL POSTGRADUATE SCHOOL

Monterey, California

AN ANALOG COMPUTER FOR USE WITH X-ARRAY HOT WIRE ANEMOMETERS

by

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January 1973

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ABSTRACT:

The design of an analog computer is presented which accurately reduces the output signals of an X-array hot wire anemometer into their basic components and calculates their auto- and cross correlations in both time averaged and instantaneous form. A prototype model is shown with certain unique features such as simultaneously available multiple outputs which give it a distinct superiority over commerically available units.

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INTRODUCTION

Because of its unique response capabilities, the hot wire anemometer is the most widely used instrument for the measurement of instantaneous turbulent or fluctuating laminar fluid flow properties. In these time-varying flows, the dependent variables of the problem are generally expressed analytically as the sum of a time independent or slowly varying mean term plus a time dependent fluctuating term. In particular, the velocity field may be decomposed as follows

$$\vec{V} = \overline{\vec{V}} + \vec{V}'(t) \tag{1}$$

For a two-dimensional mean flow we may write

$$\vec{V} = ui + vj \tag{2}$$

and

$$u = \bar{u} + u'(t) \tag{3}$$

$$v = \bar{v} + v'(t) \tag{4}$$

Initially we are interested in retrieving the mean quantities and some measure of the fluctuating terms. One such measure is the mean square (auto correlation) of the fluctuations, e.g. for the axial velocity (u)

$$u_{ms} = \overline{u'^2} = \lim_{T \to \infty} \frac{1}{T} \int_{t-T/2}^{t+T/2} u'^2 \, dt \tag{5}$$

and similarly for all other fluctuating quantities.

In fluid flow problems, when the decomposition of the dependent variables shown in Equation (1) is applied to the following equations of motion for an incompressible Newtonian fluid

$$\nabla \cdot \vec{V} = 0 \tag{6}$$

$$\rho \frac{D\vec{V}}{Dt} = - \nabla p + \mu \nabla^2 \vec{V} \tag{7}$$

subsequent time averaging yields a system of equations for the mean flow. For a two-dimensional mean flow this system may be written

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{8}$$

$$\rho \frac{D\bar{u}}{Dt} = - \frac{\partial}{\partial x} (\bar{p} + \rho \overline{u'^2}) - \frac{\partial}{\partial y} (\rho \overline{u'v'}) + \mu \nabla^2 \bar{u} \tag{9}$$

$$\rho \frac{D\bar{v}}{Dt} = - \frac{\partial}{\partial y} (\bar{p} + \rho \overline{v'^2}) - \frac{\partial}{\partial x} (\rho \overline{u'v'}) + \mu \nabla^2 \bar{v} \tag{10}$$

Here, in addition to the mean square of the axial and lateral fluctuations, the cross correlation

$$\rho \overline{u'v'} \tag{11}$$

appears as an apparent shearing stress. This term must also be studied experimentally before it can be adequately modeled.

It was specifically to measure these quantities that the hot wire anemometer was developed. Originally built as a constant current control system, constant temperature units are presently being employed to operate throughout a much wider frequency range. In either case it is the flow cooling the hot wires which generates the electrical signal to be processed. It is approximately true that the heat transfer rate to the hot wires is dependent only on the flow velocity normal to the wire. Based on this fact, an X-array hot wire anemometer probe shown in Figure 1 can be used to retrieve a variety of quantities which describe a turbulent flow, including those which appear in the basic equations of motion.

For a flow instantaneously oriented at an angle α to the axis of the probe, wire A is cooled by a flow velocity $\vec{V}\sin(\alpha + \theta)$ and wire B by a flow velocity $\vec{V}\cos(\alpha - \theta + 90^{\circ})$. Therefore, if θ is set equal to 45°, wire A responds to $u + v$ and wire B responds to $u - v$. Although the electrical response is a nonlinear function of velocity, it fits a power law referred to as King's law and can be approximately linearized by a square rooting operation. After linearization, the outputs of the hot wires can be written

$$e_A = C_A (u + v) \tag{12}$$

$$e_B = C_B (u - v) \tag{13}$$

where C_A and C_B are constants.

When $v = 0$, e_A is set equal to e_B so that $C_A = C_B = C$. If we take the time average of Equations 12 and 13 we get

$$\bar{e}_A = C (\bar{u} + \bar{v}) \tag{14}$$

$$\bar{e}_B = C (\bar{u} - \bar{v}) \tag{15}$$

so that the velocity components can be retrieved by the electrical processing

$$\bar{u} = \frac{1}{2C}\,(\overline{e_A + e_B}) \tag{16}$$

$$\bar{v} = \frac{1}{2C}\,(\overline{e_A - e_B}) \tag{17}$$

Subtracting Equations 16 and 17 from 12 and 13 we obtain

$$e'_A = C\,(u' + v') \tag{18}$$

$$e'_B = C\,(u' - v') \tag{19}$$

and the fluctuating velocities become

$$u' = \frac{1}{2C}\,(e_A + e_B)' \tag{20}$$

$$v' = \frac{1}{2C}\,(e_A - e_B)' \tag{21}$$

Clearly $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{u'v'}$ are available with simple numerical operations. Generally, however, only one output is available on most hot wire signal processors and $\overline{u'v'}$ must be obtained as follows in a two step process

$$\overline{u'v'} = \frac{1}{4C^2}\,(\overline{e'^2_A} - \overline{e'^2_B}) \tag{22}$$

DISCUSSION

With commerically available correlators, the quantities $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{u'v'}$ are obtainable with the use of RMS meters in the manner described in the previous section. The data retrieval in this way has a variety of shortcomings which can be traced to the fact that, in general, only one output u' , v' , e'_A , or e'_B can be obtained at one time and certainly never more than two. In addition, the opportunity for spectral analyses is severly restricted. To alleviate these difficulties and provide the availability of other pieces of processed data, an analog computer (Figure 3) was designed and built which generates the following simultaneous outputs

$$u \quad \bar{u} \quad u' \qquad v \quad \bar{v} \quad v' \qquad v'^2 \quad \overline{v'^2} \qquad u'v' \quad \overline{u'v'} \qquad u'^2 \quad \overline{u'^2}$$

In addition to these outputs, additional plug-in modules also shown in Figure 3 allow for the retrieval of the following triple correlations

$$u'^3 \quad \overline{u'^3} \qquad u'v'^2 \quad \overline{u'v'^2} \qquad u'^2v' \quad \overline{u'^2v'} \qquad v'^3 \quad \overline{v'^3}$$

If on line root mean squaring is specifically required, the auxiliary modules also can reduce the mean square data to root mean square data. Finally, the modules provide the capability for measuring flow angle tangents in any of the following forms

$$\frac{v}{u} \qquad \frac{\bar{v}}{\bar{u}} \qquad \frac{v'}{u'}$$

The actual design of the analog system based on Equations 14 and 15 is the essence of simplicity. Signals from each wire of an X-array hot wire anemometer probe are fed into summing and differencing amplifiers as shown in the schematic of Figure 2. Note that if the signal is not linearized, logarithmic operational amplifiers will linearize the signal here. With u , v thus obtained (within a scaling factor which may be set where indicated) the signals are broken down into $\bar{u}$, u' , $\bar{v}$, and v' by averaging and subtracting units. This approach was proved to be more efficient than the use of high pass filters to obtain

u' and v'. The fluctuating quantities are then squared and multiplied to obtain u'^2, v'^2 and $u'v'$ and finally averaged to obtain $\overline{u'^2}$, $\overline{u'v'}$, $\overline{v'^2}$. The external module then performs those additional functions which the user desires; triple correlations, RMS, and flow angles are obtained simply by interconnecting the appropriate computer outputs to the module. The basic block diagram and the individual figures (5a - 5d) are essentially self-contained explanations of each part of the computer.

The accuracy of the overall data processing is a function of the quality of the modules bought and the care exercised in the synthesis of the overall unit. Each circuit board is removable and can be calibrated for accuracy independently. The extremely high input impedances of each board allows the unit to act as a sum of the individual units without any deterioration in accuracy.

When fixed value components do not allow the linear units to perform satisfactorily, calibration potentiometers are used liberally and adjusted to give the desired output with known inputs. On the nonlinear units, some knowledge of their inherent transfer function is necessary. For example, the multiplier used in the prototype has an error map which gives the optimum ranges of input values. Based on this, the final differencing amplifiers on the computer were installed with a variable gain to increase the magnitude of the inputs to the nonlinear units. Generally, it has been convenient to scale each input by a factor of ten. Since each of the nonlinear units scale their outputs down by a factor of ten, the output of these units will be a factor of ten times the actual output value. Finally, extreme care in ground wiring and shielding has kept the noise well below anticipated levels.

Aside from the flexibility, versatility, and accuracy of this unit one of the most important features is the ability to do spectral analyses on each of the fluctuating quantities with the aid of the band pass filter shown in Figure 4. Finally the cost of the unit is comparable with commerically available units which do a fraction of the job which this computer performs.

CONCLUSIONS

On line data reduction of the signals from a hot wire anemometer can be processed by straightforward mathematical manipulation using packaged analog modules. This approach automatically overcomes many of the inherent drawbacks of commercially available units. The extent of the processing is unlimited and the accuracy is a function of the quality of the modules and the care exercised in the synthesis of the design. Aside from the ability to construct a data system tailored to some exact specifications, the ever increasing sophistication of electronic modules allows the experimentalist to create systems unequaled by commerical units at any cost.

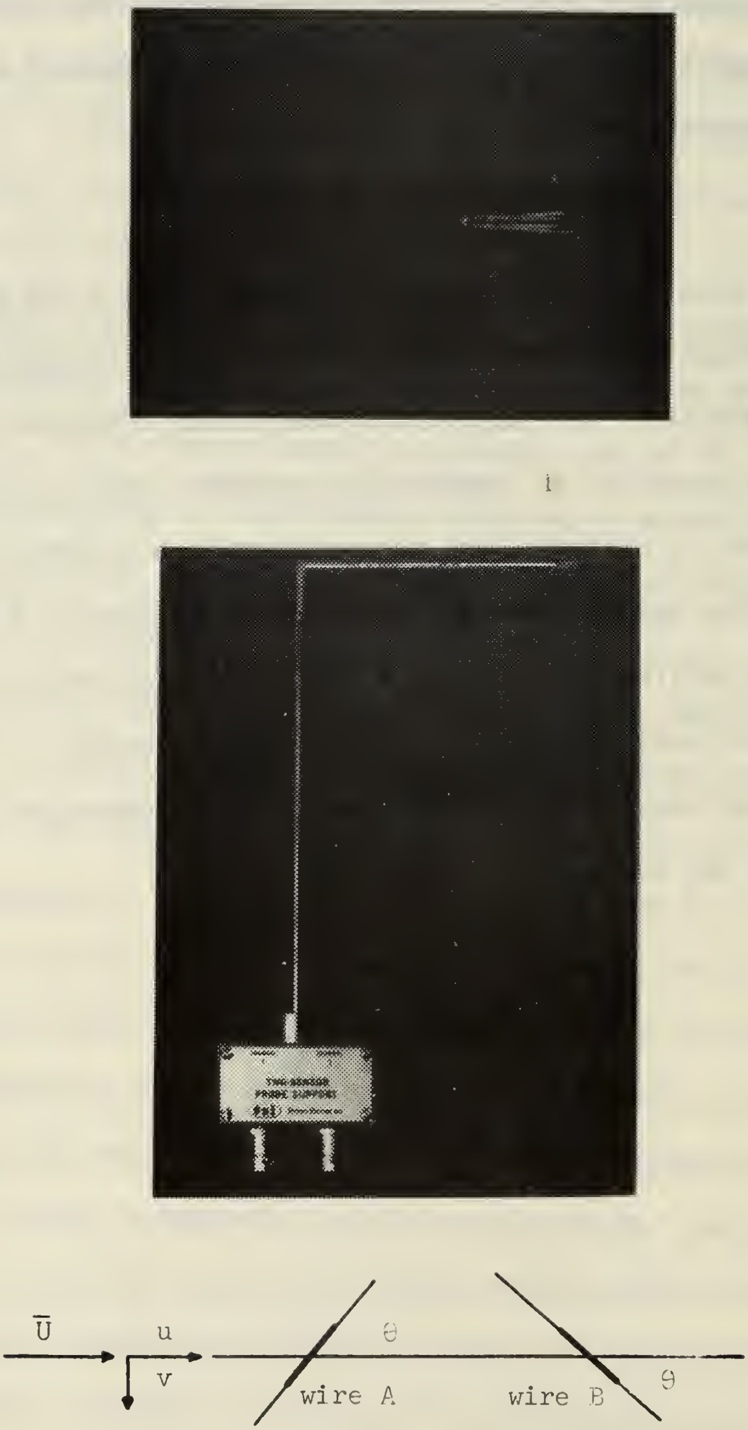


Fig. 1 X-Array Hot-Wire Probes

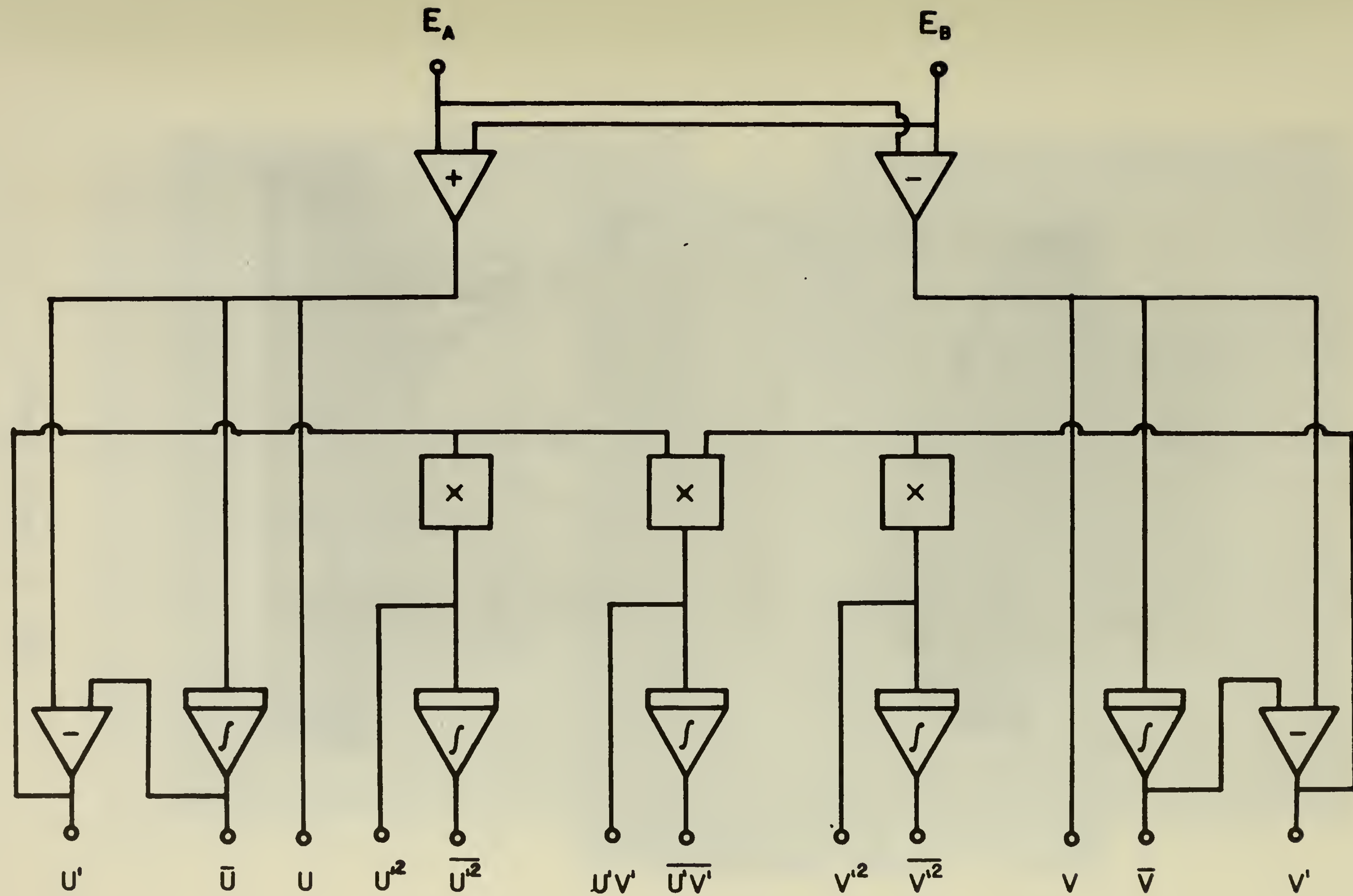


Fig. 2 Schematic of Analog Computer

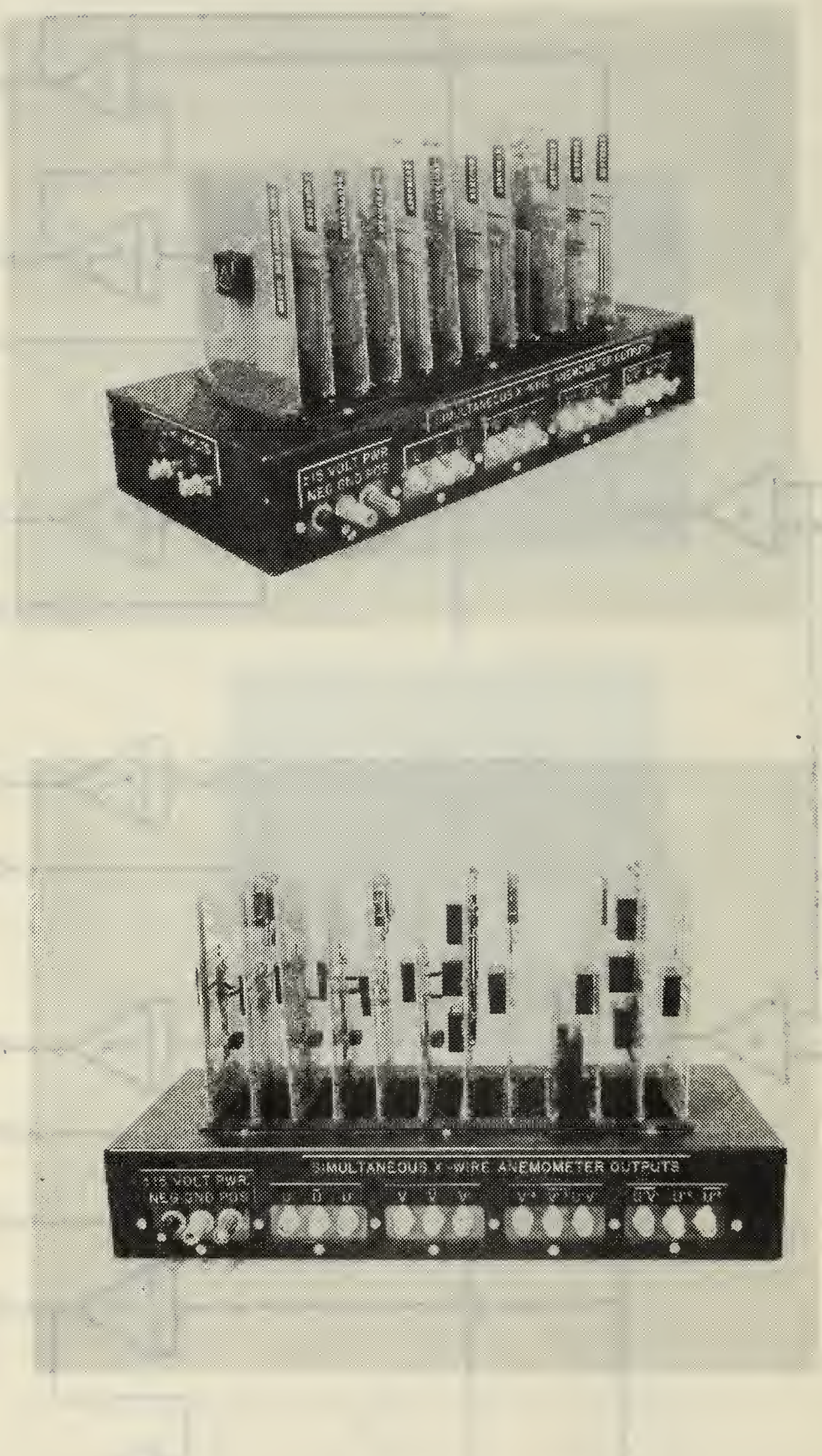


Fig. 3 Analog Computer

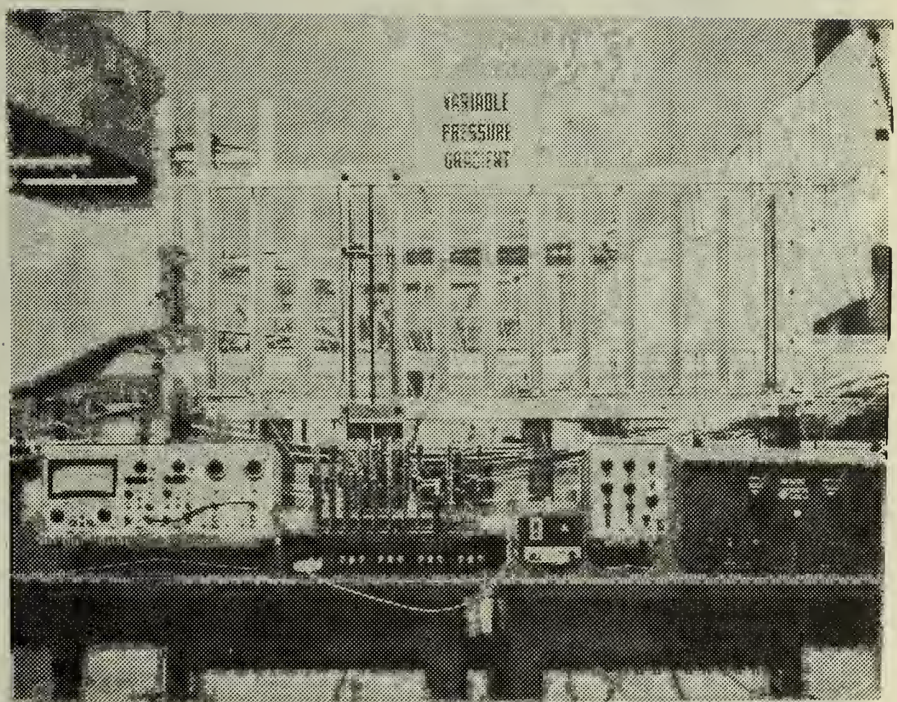


Fig. 4 Hot Wire Instrumentation

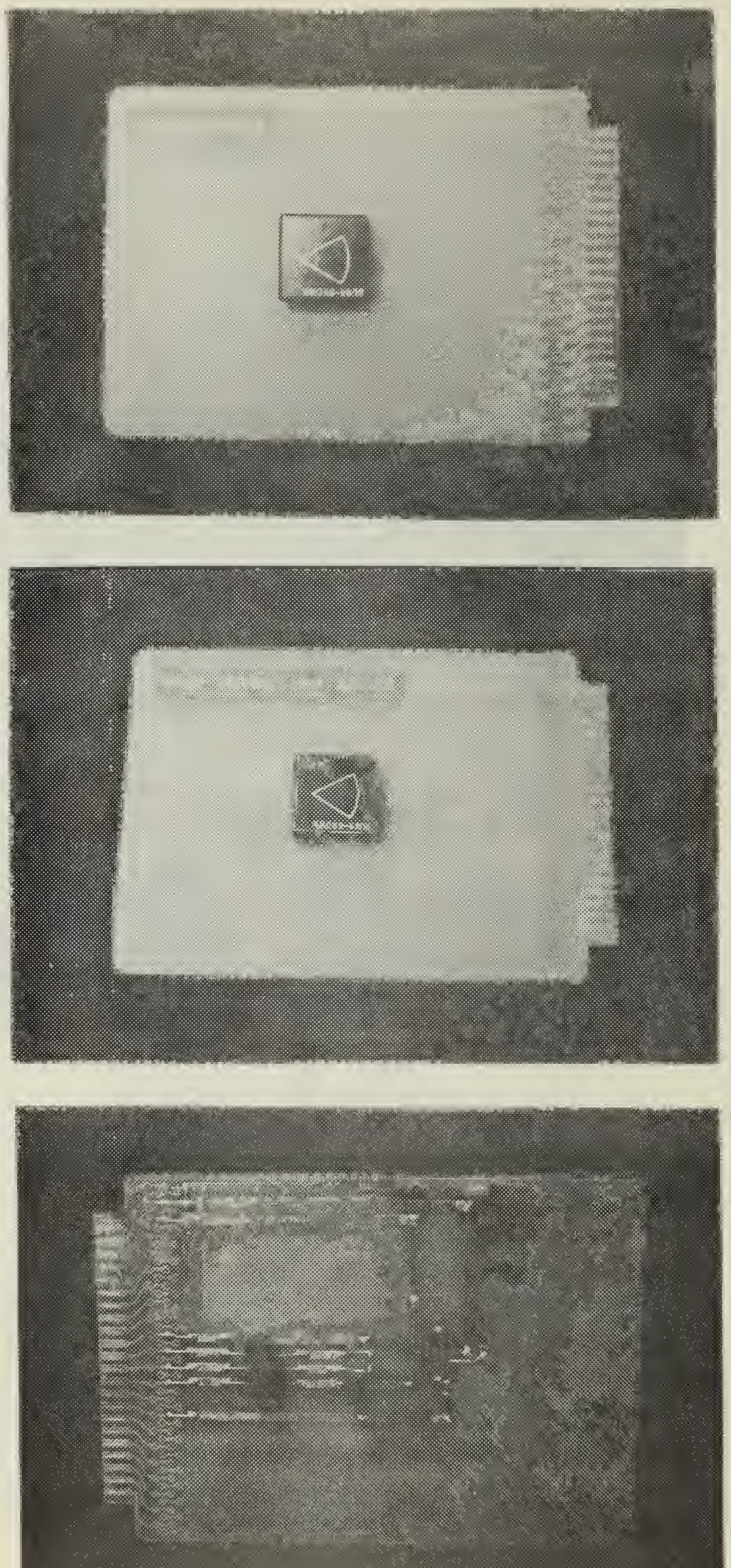


Fig. 5a Analog Summer and Differencer

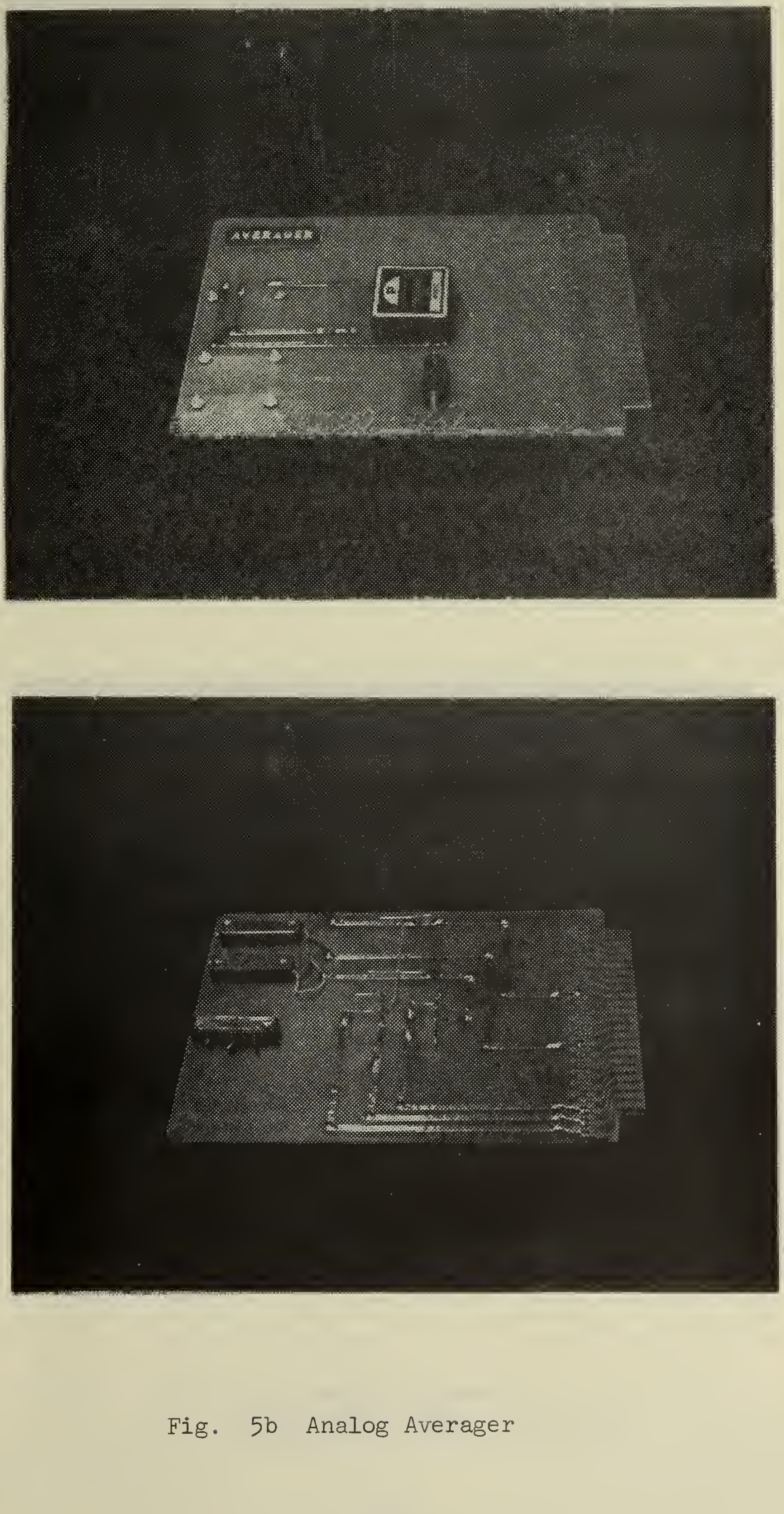


Fig. 5b Analog Averager

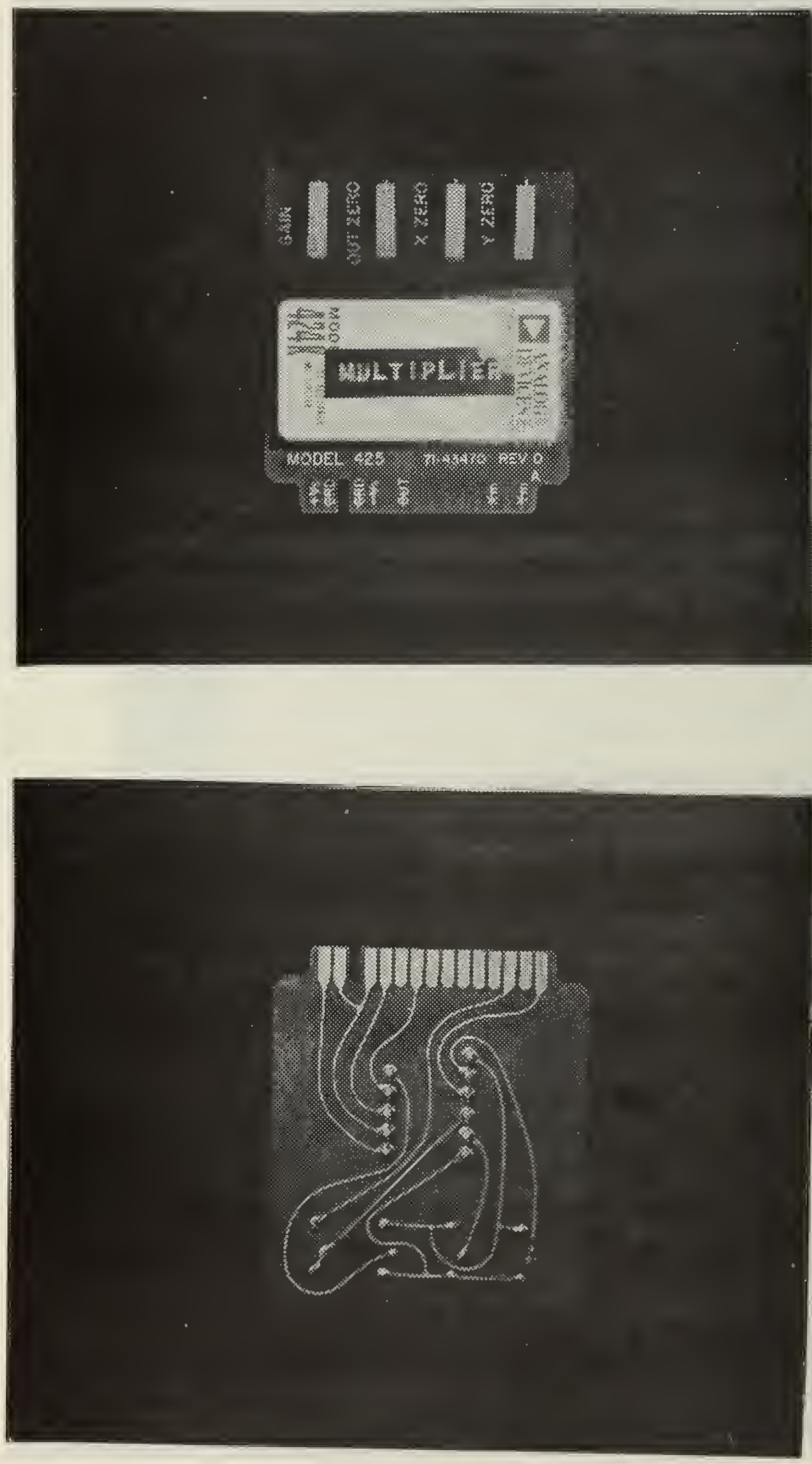
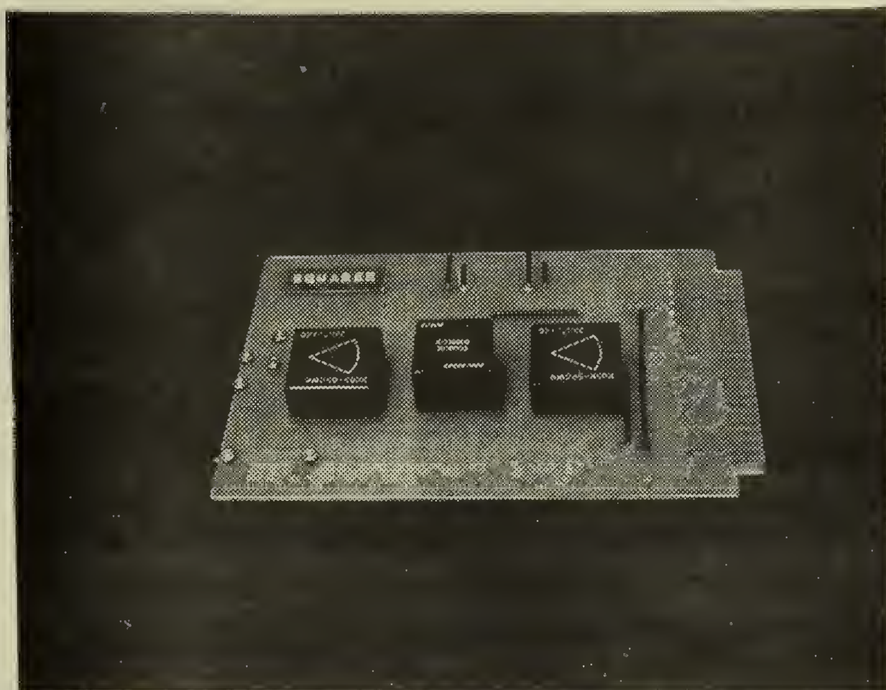


Fig. 5c Analog Multiplier



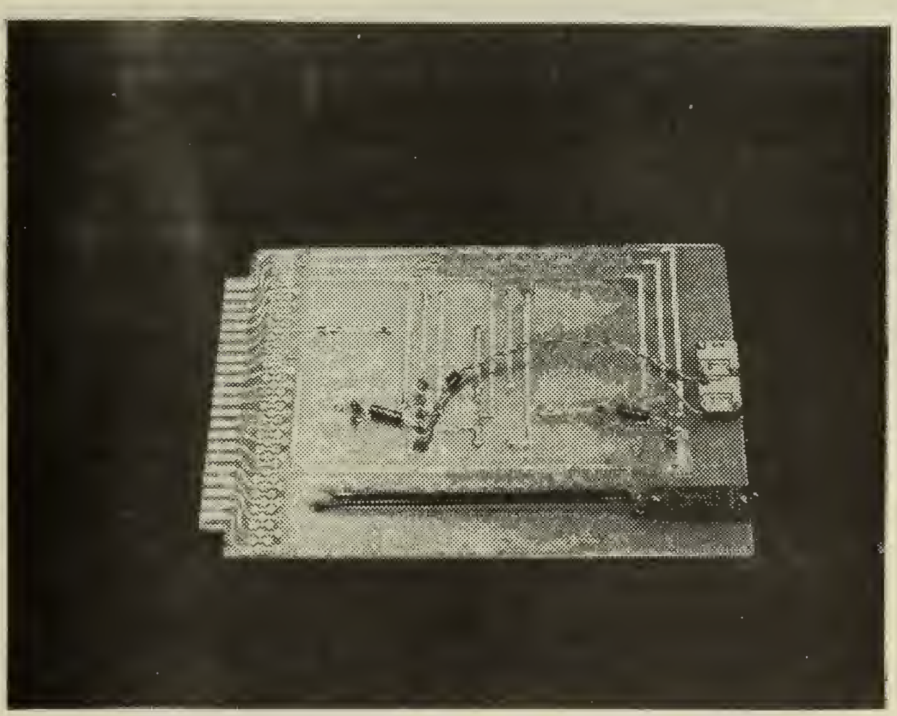


Fig. 5d Analog Squarer

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UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

An Analog Computer for Use with X-Array Hot Wire Anemometers

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

NPS-57HW73011A January 1973

5. AUTHOR(S) (First name, middle initial, last name)

Gustave Hokenson

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
	18	0

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release and sale; distribution unlimited

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey,Calif 93940

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Hot Wire Anemometry						
Analog Computer						
Instrumentation						